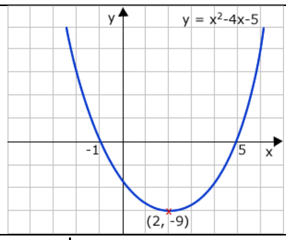
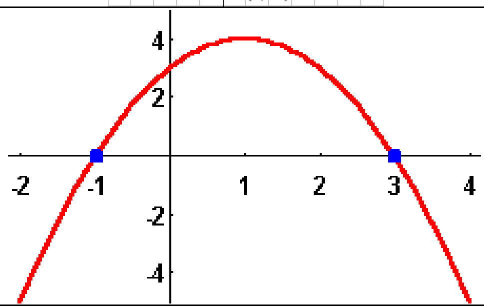
Topic: Further Quadratics

Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $$\boldsymbol{ax^2+bx+c}$$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2-3x+7$ Examples of non-quadratic expressions: $2x^3-5x^2$ $9x-1$
2. Factorising Quadratics	When a quadratic expression is in the form x^2+bx+c find the two numbers that **add to give b** and **multiply to give c**.	$x^2+7x+10=(x+5)(x+2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2+2x-8=(x+4)(x-2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2-b^2}$ can be factorised to give $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}-\boldsymbol{b})$	$x^2-25=(x+5)(x-5)$ $16x^2-81=(4x+9)(4x-9)$
4. Solving Quadratics ($ax^2=b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2=98$ $x^2=49$ $x=\pm 7$
5. Solving Quadratics ($ax^2+bx=0$)	**Factorise** and then **solve = 0**.	$x^2-3x=0$ $x(x-3)=0$ $x=0 \text{ or } x=3$
6. Solving Quadratics by Factorising ($a=1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2+3x-10=0$ Factorise: $(x+5)(x-2)=0$ $x=-5 \text{ or } x=2$
7. Quadratic Graph	A **'U-shaped'** curve called a **parabola**. The equation is of the form $y=\boldsymbol{ax^2+bx+c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a<0}$**,** the parabola is **upside down**.	
8. Roots of a Quadratic	A root is a **solution**. The roots of a quadratic are the ***x*-intercepts of the quadratic graph**.	

9. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	
10. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
11. Solving Quadratics by Factorising $(a \neq 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2}\ or\ x = -4$
12. Completing the Square (when $a = 1$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $(\boldsymbol{x} + \boldsymbol{p})^2 + \boldsymbol{q}$ 1. Write a set of brackets with x in and **half** the value of b. 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c. 4. Simplify the expression. You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x - 3)^2 - 3^2 + 2$ $= (x - 3)^2 - 7$ The minimum value of this expression occurs when $(x - 3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
13. Completing the Square (when $a \neq 1$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}(\boldsymbol{x} + \boldsymbol{q})^2 + \boldsymbol{r}$ Use the same method as above, but factorise out a at the start.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x + 1)^2 - 1^2] - 3$ $= 4(x + 1)^2 - 4 - 3$ $= 4(x + 1)^2 - 7$
14. Solving Quadratics by Completing the Square	**Complete the square** in the usual way and **use inverse operations to solve**.	Solve $x^2 + 8x + 1 = 0$ Answer: $(x + 4)^2 - 4^2 + 1 = 0$ $(x + 4)^2 - 15 = 0$

		$(x+4)^2 = 15$ $(x+4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
15. Solving Quadratics using the Quadratic Formula	A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula: $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ Use the formula if the quadratic does not factorise easily.	Solve $3x^2 + x - 5 = 0$ Answer: $a = 3, b = 1, c = -5$ $x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \frac{-1 \pm \sqrt{61}}{6}$ $x = 1.14\ or - 1.47\ (2\ d.p.)$